Practical Defeasible Reasoning for Description Logics

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Abstract. The preferential approach to nonmonotonic reasoning was consolidated in depth by Krause, Lehmann and Magidor (KLM) for propositional logic in the early 90's. In recent years, there have been efforts to extend their framework to Description Logics (DLs) and a solid (though preliminary) theoretical foundation has already been established towards this aim. Despite this foundation, the generalisation of the propositional framework to DLs is not yet complete and there are multiple proposals for entailment in this context with no formal system for deciding between these. In addition, there are virtually no existing preferential reasoning implementations to speak of for DL-based ontologies. The goals of this PhD are to provide a complete generalisation of the preferential framework of KLM to the DL $\mathcal{ALC}$, provide a formal understanding of the relationships between the multiple proposals for entailment in this context, and finally, to develop an accompanying defeasible reasoning system for DL-based ontologies with performance that is suitable for use in existing ontology development settings.

Keywords. Defeasible reasoning, Description Logics, Nonmonotonic reasoning, Preferential reasoning, OWL, Protege, Exceptions

1. Introduction

The so-called *Preferential* or *KLM* approach [16] to nonmonotonic reasoning was introduced in the early 90's for propositional logic. In recent years, it has been shown that many of the desirable aspects of this approach can be generalised to certain fragments of first order logic such as the Description Logic (DL) $\mathcal{ALC}$ [11,5, 10]. This preferential generalisation to $\mathcal{ALC}$ has some attractive attributes. Firstly, it was shown to facilitate an intuitive representation of defeasible statements (defaults) [10,5]. It also allows one to draw desirable defeasible conclusions [6, Section 3] which are as satisfactory as (if not superior to) the more well-known circumscriptive approaches [3,12]. But the most attractive properties, yet, of this logic are that it has a reasoning procedure which is composed purely of classical DL decision steps [6]; the worst case computational complexity stays the same as classical $\mathcal{ALC}$ ([11], [7, Corollary 2]) and preliminary experiments show that the performance in practice is promising [6].

Despite this progress, the generalisation of preferential reasoning to the case of $\mathcal{ALC}$ (let alone more expressive DLs) is not complete. There are still various theoretical results that have not been adapted or proven for this case and thus prevents a deeper understanding of the *ranked model* [16] semantics of preferential reasoning. The results we are interested in are those that lead to simpler reductions of preferential reasoning to classical decision steps and those that lead to gains in reasoning performance. Our hopes are that this investigation will also help to develop a deeper understanding of the relationship between defeasible KBs [5] ($\{C_1 \mathrel{\underset{\sim}{\sqsubset}} D_1, C_2 \mathrel{\underset{\sim}{\sqsubset}} D_2, ..., C_n \mathrel{\underset{\sim}{\sqsubset}} D_n\}$) and their classical counterparts ($\{C_1 \sqsubseteq D_1, C_2 \sqsubseteq D_2, ..., C_n \sqsubseteq D_n\}$) which in turn would help in building defeasible reasoning systems and related tools that are intuitive and efficient to use for ontology development. In terms of entailment, in the context of KLM preferential reasoning there are already several proposals. The consensus is that each of these proposals are suitable in different contexts. One of the aims of this PhD is to determine the relationships between these proposals and to develop an understanding of which applications each proposal is most suitable for.

We first give a preliminary introduction to preferential reasoning in DLs in Section 2. Thereafter, we mention some gaps in the theoretical understanding of preferential reasoning for DLs and the resulting barriers to developing simple and efficient algorithms thereof. The main contributions of this PhD are to address these specific issues: (*i*) to give a complete model-theoretic account of KLM-style preferential reasoning for $\mathcal{ALC}$, (*ii*) to determine the relationships between the different entailment proposals (hopefully discovering novel alternatives that are useful as well) and (*iii*) to apply the theoretical foundation in developing efficient algorithms for computing preferential inference on-demand.

2. Preferential Reasoning

In classical DLs [1], the semantics is built upon first order interpretations. These interpretations vary on the elements which appear in the interpretation domain ($\Delta^{\mathcal{I}}$) and the manner in which we assign terms (defined by an interpretation function ($\cdot^{\mathcal{I}}$)) to these elements. In the preferential context, we introduce an additional component on which the interpretations can vary. This component represents the manner in which we order the elements of the domain, using a *partial* ordering ($\prec_{\mathcal{I}}$). Interpretations with this additional component are known as *preferential* interpretations. In order to be able to *rank* the elements of our domain, we need to specify that the partial order be *modular* [5, Definition 1]. This is so that we are able to assign suitable *ranks* to elements that are incomparable in the partial order. Hence, preferential interpretations whose orderings are modular are known as *ranked* interpretations. The ordering component of a ranked interpretation allows one to interpret so-called *defeasible* subsumption statements of the form $C \mathrel{\underset{\sim}{\sqsubset}} D$ (see Figure 1).

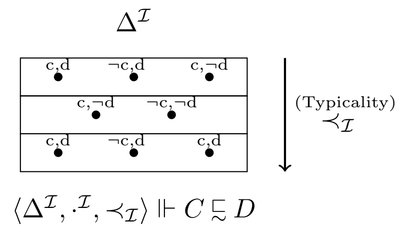


Figure 1. Satisfaction of a defeasible subsumption by a ranked interpretation.

In contrast to standard DL subsumption ($C \sqsubseteq D$), which we read as "all C's are D's", the corresponding defeasible subsumption ($C \sqsubsetsim D$) is read as "the most *typical* C's are D's". It is the ordering on the elements in a ranked interpretation that allows us to identify or specify these typical elements under consideration. The semantic paradigm which this approach captures is very intuitive because it is one which we as humans often employ (albeit in an implicit way). Consider the following example:

Example 1 *Suppose that* Bob *and* John *are mechanics. If we don't have any other information then as humans we may implicitly regard* Bob *and* John *as* typical *mechanics and assign to them properties that a typical mechanic may possess. For example we may conclude that* Bob *and* John *both work in a workshop. However, we may later discover that, while* Bob *works from a workshop,* John *is actually a* mobile *mechanic and only repairs machinery at the clients premises - which means he does not work from a workshop. One may say that* Bob *is more typical than* John *w.r.t. the property of possessing a workshop. Conversely, what this means is that* John *is more* exceptional *than* Bob *w.r.t. the same property. But what if we consider a different property of a typical mechanic? We may consider a typical mechanic to have one or more types of machinery that they* specialise *in. If we find that* John *indeed has a specialisation in motorboats but that* Bob *does not have a specialisation in any specific equipment types then we implicitly consider* John *to be more typical than* Bob *in this context.*

Example 1 demonstrates the need to consider *all* typicality orderings possible when constructing ranked interpretations of the knowledge we are reasoning about. We argue that in previous presentations of the preferential approach for DLs, there has not been enough clarity on how the approach deals with or combines *multiple* typicality orderings (as in Example 1). In Example 1 if we *only* have the constraint that typical mechanics work in a workshop then John has to be considered more exceptional than Bob in *any* ranked model thereof. Conversely, if we *only* have the constraint that typical mechanics have a specialisation then Bob is more exceptional than John. But what if we have to satisfy *both* constraints? Suppose our background knowledge is that typical mechanics work in workshops *and* that typical mechanics have at least one specialisation. Consider three of the ranked models of this knowledge in Figure 2.

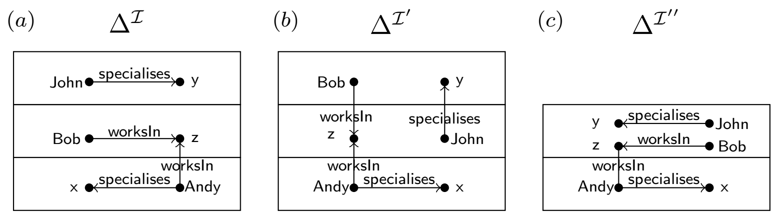


Figure 2. Combining typicality orderings using ranked interpretations.

It is clear that if our background knowledge about mechanics is correct, then there must exist at least one typical mechanic out there who satisfies both our constraints. If there isn't then we obviously have to revise or retract our statements. Since Example 1 makes mention only of Bob and John, and both these individuals are missing one of the required properties, we have to conclude that there must be a third individual. We call him Andy and he is a very typical mechanic i.e. he possesses both required properties by working in a workshop and specialising in automobiles. Both Bob and John can then be seen as exceptional w.r.t. the prototypical mechanic Andy. But how do we decide who is more exceptional between Bob and John? The answer is that we don't have to because Andy satisfies our knowledge; Bob and John are exceptional to Andy so the exceptionality distinction between them does not matter ((a), (b) and (c) in Figure 2 are all valid models of our knowledge).

A strong advantage of preferential logics is the behaviour represented in Figures 1 and 2 where the ranked interpretations satisfy that the most typical C's (lowest in the ordering) are also D's, but still allows some C's that are not as typical (higher up in the ordering) to not be D's. This is the ability to gracefully cope with *exceptions* - which is something that standard DLs cannot. We find in many fields such as biology and medicine that it is very common to encounter information which holds in general but is fallible under exceptional circumstances. Given this setting, biologists and medical professionals still have to draw conclusions and make decisions based upon this information. Preferential DLs are developed for applications of this kind.

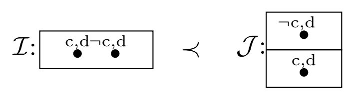
The state of the art within this framework of ranked interpretations is that we are able to reason with the terminological part of a *defeasible KB* [6] i.e. not yet with ABoxes. A defeasible KB is composed of a classical $\mathcal{ALC}$ TBox $\mathcal{T}$ and an $\mathcal{ALC}$ *DBox* $\mathcal{D}$ (set of defeasible inclusions of the form $C \mathrel{\sqsubseteq\!\!\!\sim} D$).

Given a defeasible KB $\langle \mathcal{T}, \mathcal{D} \rangle$, the obvious first proposal for entailment of a defeasible inclusion $C \mathrel{\sqsubseteq\!\!\!\sim} D$ would be to check in every ranked interpretation that satisfies every axiom in $\mathcal{T}$ and $\mathcal{D}$ and verify if $C \mathrel{\sqsubseteq\!\!\!\sim} D$ is also satisfied there (a similar approach is used for entailment in standard DLs). However, it turns out that this proposal induces an entailment relation which is *monotonic* [4, Section 4] and defeats the purpose of our logic, which is supposed to enable the representation of potentially fallible statements that can be refuted upon the discovery of new information.

But, even though the proposal to consider *all* ranked models fails as mentioned above, it is still possible to narrow our view to a subset of these. The problem is that deciding which subset to focus on may be perceived as a subjective choice. Fortunately, in the context of propositional logic, KLM have argued extensively that it is not entirely subjective [16,14]. They delineated a series of logical properties that any nonmonotonic consequence relation should satisfy at bare-minimum [16, Section 2.2]. They also pinpointed the smallest relation satisfying these properties coined the *Rational Closure* (RC) [16, Section 5].

A model-theoretic account of RC was also given by them which corresponds to considering the *minimal* ranked models [16, Section 5.7] as the base proposal for entailment. Minimal ranked models are defined by placing a partial ordering on the ranked models of the KB - this is in *addition* to the partial ordering on the elements of the domain (see Figure 3 for an example).

$$\langle \mathcal{T}, \mathcal{D} \rangle = \langle \emptyset, \{ C \underset{\sim}{\sqsubset} D \} \rangle$$



$\mathcal{I}$ is a minimal ranked model for $\langle \mathcal{T}, \mathcal{D} \rangle$

Figure 3. Ordering ranked models in pursuit of the minimal ones.

The minimal ranked models in the partial order are those in which there is no element of the domain that can be moved to a more typical level in the strata (i.e. if it can be moved, then it is not possible without violating at least one axiom in the KB).

The logical properties that any nonmonotonic consequence relation should satisfy were shown to generalise well to the DL case ([4, Definition 4] and [5, Definition 2]). Several DL generalisations of RC have also been proposed [7,10,5,4]. Giordano et al. [10] gave the first generalisation of RC which corresponds in a natural way to the minimal ranked model semantics of KLM [16]. Our characterisation [4] was also shown to correspond to theirs.

The first attempt at a procedure for computing RC in the DL case was the effort of Casini and Straccia [7] for $\mathcal{ALC}$. This syntactic procedure was composed entirely of classical DL decision steps. A tableau calculus was presented for a preferential extension of $\mathcal{ALC}$ by Giordano et al. [11]. Notwithstanding, all existing procedures in the literature that are based on classical DL decision steps are variants of the syntactic procedure by Casini and Straccia [7].

The full technical details of our procedure including pseudocode has been presented [6]. We conclude our brief survey of preferential reasoning in DLs with an example illustrating the kinds of inferences we can draw with RC, the limitations of RC (the inferences that we would like to draw but cannot), and the additional inferences we can draw from recent extensions of RC such as the Lexicographic [15,8] and Relevant closures (submitted work).

Example 2 *Consider the following defeasible KB:*

$$\mathcal{T} = \left\{ \begin{array}{ll} 1.\ \mathsf{MRBCell} & \sqsubseteq \mathsf{ECell}, \\ 2.\ \mathsf{HRBCell} & \sqsubseteq \mathsf{MRBCell}, \\ 3.\ \mathsf{CamelRBCell} & \sqsubseteq \mathsf{MRBCell}, \\ 4.\ \exists hasShape.\mathsf{Circle} & \sqsubseteq \neg\exists hasShape.\mathsf{Oval} \end{array} \right\}$$

$$\mathcal{D} = \left\{ \begin{array}{ll} 1.\ \mathsf{ECell} & \precsim \exists hasNucleus.\top, \\ 2.\ \mathsf{MRBCell} & \precsim \neg\exists hasNucleus.\top, \\ 3.\ \mathsf{MRBCell} & \precsim \exists hasShape.\mathsf{Circle}, \\ 4.\ \mathsf{HRBCell} \sqcap \exists hasCondition.\mathsf{EMH} & \precsim \exists hasNucleus.\top, \\ 5.\ \mathsf{CamelRBCell} & \precsim \exists hasShape.\mathsf{Oval} \end{array} \right\}$$

*The KB consisting of $\mathcal{T}$ and $\mathcal{D}$ above represents biological information describing that: eukaryotic cells (*ECell*) usually have a nucleus, mammalian red blood cells (*MRBCell*) are types of eukaryotic cells that usually don't possess a nucleus, human red blood cells (*HRBCell*) are also mammalian red blood cells but if they are affected by the* extramedullary hematopoiesis *[18] (*EMH*) medical condition then they usually contain a nucleus. In addition to the properties pertaining to nuclei, we also represent that mammalian red blood cells generally have a circular shape but the red blood cells of a camel (*CamelRBCell*), which are also mammalian, do not inherit this property (they are distinctly oval shaped) [17].*

Using RC we are able to derive (retain) the intuitive inferences that: eukaryotic cells usually have a nucleus and even though mammalian red blood cells are considered eukaryotic, they are allowed to break the tradition and be devoid of a nucleus. In essence, mammalian red blood cells are recognised by RC as exceptional *eukaryotic cells. RC also caters for* exceptions to exceptions *by noting that a human red blood cell that is infected with* EMH *is an exceptional mammalian red blood cell and is therefore allowed to possess a nucleus.*

However, a limitation of RC is that it will not draw the reasonable inference that: human red blood cells (even if they are infected with EMH*) should be circular in shape [15,8]. We can argue that this inference is reasonable to make because we know that mammalian red blood cells usually have a circular shape (Axiom* 3 *in $\mathcal{D}$), and that human red blood cells are mammalian (Axiom* 2 *in $\mathcal{T}$). The trouble is that RC sees human red blood cells with* EMH *as exceptional even though the reason for this has nothing to do with its shape (the reason is related to the property of possessing a nucleus). Together with the fact that RC does not permit inheritance of properties for exceptional elements, the desired inference is not allowed. In an analogous way, we cannot derive another desirable conclusion that a camel red blood cell should not possess a nucleus.*

The Lexicographic and Relevant closures are syntax dependent *extensions of RC that overcome the above limitations [15,8]. They do this by identifying the reasons for information to be considered exceptional in the KB (albeit in different ways). Relevant closure (submitted work) notably uses the notion of justifications [13,2] in this regard which further exploits the connection between nonmonotonic reasoning and belief revision [9]. In both these proposals, we are able to* *derive from Example 2 that human red blood cells infected with* EMH *are usually circular in shape and that camel red blood cells usually lack a nucleus.* □

3. Open Issues

As mentioned earlier, the framework for preferential reasoning in DLs is not complete. "Lifting" the theoretical results from the propositional case to the DL case is not straightforward in all situations. For example, there is a definition in the propositional case for the exceptionality of a formula w.r.t. to a defeasible KB [16, Definition 2.20]. We give the natural translation of this definition for $\mathcal{ALC}$:

Definition 1 *We define an $\mathcal{ALC}$ concept, C, as being* exceptional *w.r.t. a defeasible $\mathcal{ALC}$ KB $\langle \mathcal{T}, \mathcal{D} \rangle$ if $\langle \mathcal{T}, \mathcal{D} \rangle \models_p \top \sqsubsetsim \neg C$. Each $C \sqsubsetsim D \in \mathcal{D}$ is also said to be* exceptional *w.r.t., $\langle \mathcal{T}, \mathcal{D} \rangle$.*

Definition 1 uses ranked entailment ($\models_p$) to define the exceptionality of a concept C. Intuitively, it says that C is considered exceptional w.r.t. the KB if the most typical elements of the domain cannot belong to the extension of C in any ranked model of the KB. In other words, from the information in the KB, it is abnormal to belong to the extension of C. This definition is quite straightforward and intuitive to understand in terms of ranked models but there is no relationship drawn to help us understand this in terms of classical DL interpretations. More specifically, there is no straightforward reduction of exceptionality (from the ranked entailment notion described above) to some form of classical entailment. We argue that this would be useful for a variety of reasons. For one, it would deepen our understanding of the relationship between defeasible KBs and their corresponding classical counterparts, and secondly, it would help in developing more optimised algorithms for computing preferential reasoning.

Another issue that needs addressing is the fact that there are several alternatives to answer the question of entailment. Rational Closure, is deemed the appropriate starting point since it is the most conservative relation satisfying KLMs logical postulates [16, Section 2.2]. But for the growing number of alternatives to RC including Lexicographic Closure, Relevant Closure and even non monotonic reasoning proposals outside the KLM framework, there needs to be an investigation into exactly how they relate to RC and the logical postulates. We plan to investigate these relationships in terms of the entailments that they give, the applications where each is most suitable and their reasoning performance.

Finally, the ultimate goal is to enable the practical use of preferential reasoning in ontology development settings where DLs are the main underlying formalism. The OWL (`www.w3.org/TR/owl-features`) standard and OWL-related tools for ontology development are the main targets for the introduction of preferential reasoning features. Optimisations are needed to enable on-demand reasoning in OWL tools. In addition, various new avenues for research open up when considering non-standard reasoning services in the preferential context. Tasks such as classification (computing the subsumption relationship between each pair of concept names in the ontology) and axiom pinpointing [2] cannot be translated to the preferential context in a trivial way.

4. Conclusion

We have presented a general outline of the research that will be conducted for this PhD. The ultimate goal is to enable on-demand reasoning services for DL-based ontologies that represent defeasible subsumptions in the preferential framework. Before that goal can be achieved we propose to lift the solid theoretical foundation that was established by KLM, in propositional logic, to the DL $\mathcal{ALC}$. The different entailment proposals have to be documented and placed into perspective w.r.t. each other. These need to be compared and evaluated to determine their suitability in different contexts and practical performance. Finally, we plan to optimise the proposals and implement them in a defeasible reasoning system that can be integrated into existing OWL-related tools and systems.

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